2
THEORIES AND MODELS

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1. Introduction

There are models, and there are theories. This invites the question of how the two are related. Traditionally, it was assumed that this question had a simple answer, and attempts have been made to explain the relation between models and theories at a general level. In this chapter, I argue that there is no such thing as "the" relation between models and theories. How models relate to theories depends on the cases at hand, and models can stand in a multiplicity of relations to theories.

The chapter starts with a discussion of the Syntactic View and Semantic View of theories and points out that these views have too narrow a vision of what models are and of how they relate to theories (Section 2). We then discuss different relations between models and theories in descending order of models' independence from theory. We begin by looking at models that are constructed without the aid of a theoretical framework and that therefore end up being independent from theory (Section 3). An interesting class of models serves the purpose of exploring the properties of a theory by providing simplified renderings of a theory's features (Section 4). In some cases, models live in a symbiotic relation with theories, adding specifics about which the theory remains silent (Section 5). In other cases, the reliance of theories on models is even stronger because theories require interpretative and representative models in order to relate to real-world targets (Section 6), which motivates the view that models are mediators between theories and the world (Section 7). Sometimes it is difficult to draw the line between models and theories, and we discuss how, and where, such a line could be drawn (Section 8). Section 9 concludes.[1]

2. Two orthodoxies

Twentieth-century philosophy of science has produced two broad views of what scientific theories are, and both imply a position on how models relate to theories. For better or worse, these two views form the backdrop of most discussions of models and theories today, and so our discussion should begin with them.

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The first view, often referred to as the *Syntactic View of Theories* ("Syntactic View", for short), is associated with logical empiricism. Early statements of the Syntactic View include Carnap (1923) and Schlick (1925); full developments can be found in Carnap (1938, sec. 23), Braithwaite (1953, chaps. 1–3; 1954), Nagel (1961, chap. 5), and Hempel (1966, chap. 6; 1970).[2] The Syntactic View regards a theory T as a linguistic entity that satisfies the following three requirements:

(R1) T is formulated in an appropriate system of formal logic.
(R2) T contains axioms, which, when interpreted, are the theory's laws.
(R3) T's extralogical terms are divided into observation terms and theoretical terms, and theoretical terms are connected to observation terms by correspondence rules.

R1 is often said to mean that the theory is formulated in first-order predicate logic, but this restriction is unnecessary and T can be formulated in any system of logic (Lutz 2012). R2 requires there to be general propositions in the logical system which are the theory's laws when the extralogical terms are given an empirical interpretation. As a simple example, consider the sentence $(\forall x)(Fx \rightarrow Gx)$. Taken on its own, this is just a formal sentence (saying that for every object x, if x has property F, then x also has property G). This sentence becomes a statement of a law of nature of a simple theory of electricity if we interpret F as "is a piece of copper" and G as "conducts electricity". Under this interpretation, the sentence says that every object that is a piece of copper also conducts electricity. R3 harbours the view's empiricist commitments. Extralogical terms are terms that relate to objects and properties in the world (in contrast to logical terms like "and" and "or", which concern the structure of sentences). The Syntactic View separates these into observation terms and theoretical terms. The former are terms like "round", "green", "ball", "liquid", "wheel", "hot", "longer than", and "contiguous with", which refer to directly observable objects, properties, and relations. The latter are terms like "electron", "entropy", "orbital", "electromagnetic field", "gene", "quantum jump", "temperature", and "rate of inflation", which (purportedly) refer to objects, properties, and relations beyond direct observation. The view postulates that theoretical terms are related to observation terms by so-called *correspondence rules*. By way of illustration, consider "temperature". The temperature of an object is not directly observable. What is observable are thermometer readings. So the Syntactic View postulates that the term "temperature" be connected to an observation term through a rule like "an object has temperature θ if, and only, a thermometer shows θ when brought in contact with the object".[3]

Let us call the theory's system of formal logic together with its uninterpreted axioms the theory's formalism. The formalism of a theory is a set of formal sentences. Given such a set of sentences, one can always look for a set of objects, along with their properties and relations, which make the sentences true if the sentences' terms are interpreted as referring to those objects, properties, and relations. Such a set of objects constitutes a *logical model*. It is then common to say that the model *satisfies* the formal sentences in the sense that the model makes the sentences true if the terms of the sentences are taken to refer to the objects, properties, and relations in the model. In the context of a discussion of scientific theories, the relevant formal sentences are stated in the language of the formalism of a theory, and hence logical models are sometimes referred to as "models of a theory" or "models for a theory".

If, for the sake of illustration, we assume that the formalism of our theory consists only of the sentence $(\forall x)(Fx \rightarrow Gx)$, then a set of objects is a model for that theory if it is the case that to every object to which the predicate *F* applies, the predicate *G* also applies. Earlier we interpreted *F* as "is a piece of copper" and *G* as "conducts electricity". But interpretations are not unique, and formalisms can often be interpreted in several different ways. Rather than interpreting *F* and *G* in terms of copper and conductivity, we could interpret *F* as "is a piece of granite" and *G* as "contains quartz", which also makes the sentence $(\forall x)(Fx \rightarrow Gx)$ true. Hence, a set of objects in which it is the case that every object to which "is a piece of granite" applies is such that also "contains quartz" applies to it is a model of the theory.

In the Syntactic View, *scientific models* are essentially alternative interpretations of a theory's formalism. Braithwaite expresses this clearly when he says that a model is "another interpretation of the theory's calculus" (1962, 225), whereby his "calculus" is synonymous with our "formalism". However, for an alternative interpretation to be useful, it must have an additional feature: the objects of the alternative interpretation must be familiar to us. In Hesse's words, "a model is drawn from a familiar and well-understood process" (1961, 21). Crucially, this requirement applies to *all* terms of the formalism. That is, it applies also to the terms that were considered theoretical terms under the standard interpretation of the theory. In R3, these terms were given an "indirect" interpretation via correspondence rules, which made them difficult to grasp intuitively. In the context of a model, these terms receive a direct interpretation based on something familiar to us. In sum, then, we can say that according to the Syntactic View, a scientific model (often just "model") is a logical model of a theory's *entire* formalism that consists of objects, properties, and relations that are familiar to us.

As an example, consider the kinetic theory of gases. The theory takes a gas to consist of molecules that move freely unless they either collide with each other or the walls of the vessel containing the gas. Since "gas molecule" and "trajectory of a molecule" are theoretical terms, the theory is not easy to comprehend. To get an intuitive grip on the theory, we can reinterpret the theory in terms of billiard balls and their paths. The terms that were formerly interpreted as referring to molecules are now interpreted as referring to billiard balls; the terms that were interpreted as referring to the trajectories of molecules are now interpreted as referring to the paths of billiard balls. A bunch of billiard balls is therefore a model of the kinetic theory of gases. Other well-known examples of models of this kind are water waves as a model of the acoustic theory of sound waves and the solar system as a model of the Bohr theory of the atom.

The second view of theories in 20th-century philosophy of science is the so-called *Semantic View of Theories* ("Semantic View", for short). Historically this view was intended to replace the Syntactic View, which has been reported to suffer from a number of serious problems. It is a matter of controversy whether these problems are as severe as critics have said they were, or whether they are problems at all. However, this is not the place to review this debate and the reader is referred to the relevant literature on the subject.[4] Important statements of the Semantic View include Suppes (2002), van Fraassen (1980), Balzer, Moulines and Sneed (1987), Giere (1988), and Da Costa and French (1990). Different authors develop the view in different ways, but there is a common denominator, the focus on a theory's models. As we have seen previously, a logical model is a set of objects (along with their properties and relations) that make the theory's formalism true. We can then ask what the class of all logical models of a formalism looks like, and this will give us important information about the nature of a theory. Hence, rather than focussing on the formalism itself when characterising a theory, we can focus on its models. The Semantic View submits that this is not just another way of doing the same thing; on the contrary, characterising a theory in terms of its models is superior to characterising it in terms of its formalism. The primary reason for this is that formalisms can change and yet describe the same things. We are familiar with this phenomenon from everyday contexts, where we can say the same thing in different languages. "Copper conducts electricity" and "Kupfer leitet Elektrizität" are different sentences but they have the same truth-maker, namely the fact that copper conducts electricity. In the context of theories, we can choose different formal tools to describe the same models, which, however, would not result in a new theory because such reformulations merely describe the same thing in different ways. This motivates the Semantic View's core posit: a scientific theory is a family of models. For instance, in the Semantic View, Newtonian mechanics is not a set of postulates about motion and force; it is the set of models in which these postulates are true.

Two points deserve note. The first is that different authors have different ontologies of models. Suppes and Balzer, Moulines and Sneed take them to be set-theoretical structures; Da Costa and French take them to be partial structures; van Fraassen takes them to be state spaces; and Giere takes them to be abstract objects. These differences are important in other contexts, but they are immaterial to the discussion in this chapter. The second is the role of a formalism. We introduced the Semantic View by appealing to the notion of a logical model, and indeed, it is that notion that gives the view its name: the view is called the "Semantic" View due to the fact that models provide the formalism's semantics because models are what the formalism is taken to be about. Yet, providing a semantics for a formalism is like Wittgenstein's ladder, which is pushed away after it has been climbed. Proponents of the Semantic View insist that interpreting a formalism is in no way essential, nor is the presence of a formalism to begin with. At bottom, a theory is simply a family of models, no matter how (if at all) they are described by a formalism.

As indicated previously, much can be said about the pros and cons of these two views, but this is not our subject matter. What interests us here is the analysis of the relation between models and theories that the two approaches offer. The core argument of this chapter is that both analyses are too narrow. To see why and how, note that in both conceptions, models play a subsidiary role to theories. In the Syntactic View, they are merely reinterpretations of a formalism in terms of something familiar; in the Semantic View, they are the building blocks of which theories are made up. Both notions capture some cases of modelling. The Syntactic View successfully explicates analogue models, which often connect to their target via a shared formalism.[5] The Semantic View offers a cogent analysis of what happens in certain areas of fundamental physics, most notably in theories of space and time.[6] However, there are many cases, and indeed entire areas of science, where the relation between models and theories fits neither the mould of the Syntactic View nor that of the Semantic View. The plan for the remainder of this chapter is to discuss cases of this kind.

3. Models without theory

There are models that are independent of any theory. An often-discussed example of such a model is the so-called Lotka–Volterra model.[7] Volterra's version of the model is about the fish population in the Adriatic Sea. Volterra conceptualised the problem as a population-level phenomenon with a population of predators interacting with a population of prey. The populations are described solely in terms of their sizes, and no biological facts about the animals that constitute the populations are taken into account (beyond the obvious truism that predators eat prey and not *vice versa*). Let N_1 be the number of prey and N_2 the number of predators. Volterra then asked how these numbers change over time. The change in these numbers is due to intrinsic births and deaths in both populations, as well as to the interaction between the two. The general form of the interaction can therefore be expressed as follows (Kingsland 1985, 109–100):

Change in N_1 per unit of time = Natural increase in N_1 per unit of time minus decrease in N_1 per unit of time due to destruction of prey by predators

Change in N_2 per unit of time = Increase in N_2 per unit of time due to ingestion of prey by predators minus decrease of N_2 due to deaths of predators per unit of time.

These "verbal equalities" can be turned into proper mathematical equations by replacing the natural numbers N_1 and N_2 by the continuous quantities V (for the quantity of prey) and P (for the quantity of predators) and by choosing specific functions for the population growth and the interactions between the populations. The simplest choice is to assume that each population grows linearly and that the interaction between the populations (predators eating prey and growing as result) is proportional to the product of the two densities. Inputting these formal choices into the above equalities leads to the so-called Lotka–Volterra equations (Weisberg and Reisman 2008, 111):

$$\begin{aligned} \dot{V} &= rV - (aV)P \\ \dot{P} &= b(aV)P - mP, \end{aligned} \tag{2.1}$$

where r is the birth rate of the prey population; m is the death rate of the predator population; and a and b are linear response parameters. The dots on V and P indicate the first derivative with respect to time. Intuitively, $\dot{V}$ is the rate of change of V and ditto for $\dot{P}$.

Even though Volterra notes that Darwin had made an observation similar to his own (1926, 559), neither Darwinian evolutionary theory nor any other biological theory is at work in the model. Indeed, the model has been constructed without a theoretical framework, and it does not instantiate theoretical principles. As a result, the model is independent of theory.

The Lotka–Volterra model is not an isolated instance. The Schelling model of social segregation (Schelling 1978), the Fibonacci model of population growth (Bacaër 2011, chap. 1), the logistic model of population growth (May 1976), the Akerlof model of the market for used cars (Akerlof 1970), and complexity models for the behaviour of sand piles (Bak 1997) are "theory-free" in the same way. Models of this kind are sometimes characterised as bottom-up models. A model is *bottom-up* if the process of model construction departs from the basic features of the target and from what we know about the unfolding of events in the domain of interest, while not relying on general theories. Bottom-up models contrast with top-down models. A model is *top-down* if the process of model construction starts with a theoretical framework, and the model is built by working the way down from the theory to the phenomena. The Newtonian model of planetary motion is an example of a top-down model. The process of model construction starts with Newton's general equation of motion and the law of gravity, and then various steps are made to apply these general principles to the phenomenon of interest, namely the movement of planets.

A special case of models that are independent of theories are models that are built with the express aim of aiding the construction of theories. Leplin emphasises the importance of models in the construction of theories and calls models that are constructed with this purpose in mind *developmental models* (1980, 274). A developmental model "opens several lines of research toward the development" of a theory (278). The importance of models in the development of theories has also been emphasised by other authors. Cushing notes that "[a]n important tool in this process of theory construction is the use of models" (1982, 32), and he illustrates this with a detailed case study from high-energy physics. Hartmann observes that "[a]s a major tool for theory construction, scientists use models" (1995, 49), and he illustrates this with how quantum chromodynamics, the fundamental theory of strong interactions, has been constructed "by means of a hierarchy of consecutive *Developmental Models*" (59). Wimsatt, finally, sees "false models as a means to truer theories" and discusses their construction in the context of evolutionary biology (Wimsatt 2007, chap. 6).

4. Models as a means to explore theories

Models can also be used to explore the features of theories. A case in point is the study of non-linear dynamics. For a long time, it was thought that Newtonian mechanics was dynamically stable, meaning that a small variation in the initial condition of the system would result in a small variation in the trajectory of the system. This belief was shattered at the beginning of the 20th century when Poincaré discovered that Newtonian systems can display what is now known as *sensitive dependence on initial conditions*, which is often taken to be the defining feature of chaos.[8] This raises the question of how the dynamic of such systems looks like. Unfortunately, one cannot simply write down the solutions of the equations of motion of such systems and study their properties; and even if one could write down the solutions, they would be objects in high-dimensional mathematical spaces that are hard to trace and impossible to visualise. Thus, other means to understand the behaviour of such systems must be found, and models play a crucial role in this.

Abstract considerations about the qualitative behaviour of solutions in chaotic systems show that there is a mechanism that has been dubbed *stretching and folding*. Nearby initial conditions drift away from each other, which amounts to stretching the area where they lie. The motion of chaotic systems is such that the system's movement is confined to a restricted part of the state space. This means that the stretching cannot continue forever, and the stretched bits must be folded back onto each other. In practice, it is impossible to trace this stretching and folding in the full state space of a system. To obtain an idea of the complexity of the dynamic exhibiting stretching and folding, Smale proposed to study a model of the flow. The model is a simple two-dimensional map, now known as the horseshoe map (Tabor 1989, 200–202), which is illustrated in Figure 2.1.

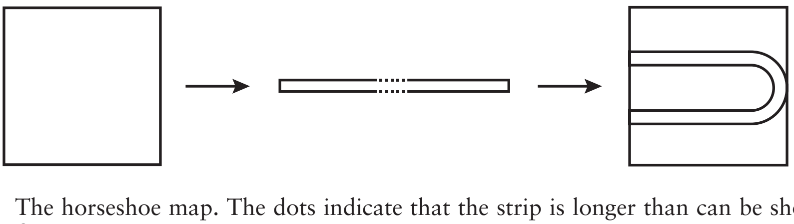

Figure 2.1 The horseshoe map. The dots indicate that the strip is longer than can be shown in the figure.

The map begins by stretching a rectangle horizontally while squeezing it vertically, which turns the rectangle into a strip; it then folds the strip back onto the initial square. The map is designed to "mimic" the stretching and folding motion of the full Newtonian dynamic, but without having any of its mathematical complexities. In this way, the horseshoe map provides a model of an important aspect of the full dynamic of Newtonian theory. The horseshoe map has a number of interesting and important features (Ott 1993, 108–114). An *invariant set* is a set of states that does not change under the dynamic of a model – it is as if the set was not "affected" by the changes that the dynamic brings with it. One can show that the so-called Cantor set is an invariant set of the horseshoe. This is interesting because the Cantor set is a fractal, and so we learn from the model that chaotic dynamical systems can have invariant sets that are fractals. In this way, the simple model of the horseshoe provides a crucial insight into the properties of the theory. The horseshoe is no isolated instance: chaos theory is rife with maps that model certain aspects of the full dynamic and thereby shed light on the nature of the theory itself.[9]

Chaos theory is no exception, and models are used in many contexts to explore the properties of theories. In statistical mechanics, the Kac ring model is used to study the equilibrium properties of the full theory (Jebeile 2020; Lavis 2008). In quantum field theory, the φ^4 model is used to explore theoretical properties like symmetry breaking and renormalisability (Hartmann 1995). The Phillips–Newlyn machine, a material model, is used to explore the properties of Hicks' formalisation of Keynes' theory (Barr 2000; Morgan and Boumans 2004). And the dome model is used to understand causality and determinism in Newtonian mechanics (Norton 2008).

5. Models complementing theories

Theories can be incompletely specified. Models can then step in and add what is missing. The model and the theory thereby enter into a symbiotic relationship in which a model complements the theory. The nature of this "completion" depends on the specifics of the case. Redhead (1980, 147) mentions the case of axiomatic quantum field theory. The theory is an attempt to offer a mathematically rigorous formulation of quantised fields. In its most common formulation, the theory is based on the so-called Wightman axioms. Roughly, the axioms say things like that fields must be invariant under the transformations of Einstein's theory of special relativity and that fields can be expressed as sums of operators acting on the vacuum state.[10] This means that the theory's axioms only impose certain general constraints on fields, and the specifics of particular fields and their interactions are given by models. In doing so, the model provides missing details and enriches the theory. This is not an easy task because it turns out that identifying models that satisfy the axioms of the theory is rather difficult.

Another way in which a theory can be incompletely specified is identified by Apostel when he notes that there are cases where "a qualitative theory is known for a field and the model introduces quantitative precision" (1961, 2). As an example, consider the so-called *quantity theory of money* in monetary economics.[11] The "quantity theory" is purely qualitative and essentially says that the price of goods in an economy is determined by the amount of money in circulation. This law leaves open what the price levels are and how they vary as a function of money supply. To answer these quantitative questions, Fisher constructed a model that is now known as *Fisher's equation of exchange*. The model considers an economy that can be characterised by four quantities: the amount M of money in circulation, the transaction velocity V of money, the level of prices P, and the volume of trade Y. All these are variables with precise numerical values that can, in principle, be measured empirically. The equation of exchange is $MV = PY$. If velocity and volume are constant, the equation says that $P = cM$, where c is a constant. So if the amount of money increases by ΔM, then prices go up by $c\Delta M$. In this way, Fisher's model gives quantitative specificity to the qualitative law of the theory.

Harré (2004) noted that models can complement theories by providing mechanisms for processes left unspecified in the theory but that are nevertheless responsible for bringing about the observed phenomena (2004, chap. 1). In some cases, the model mechanism is known; in other cases, it is hypothesised. The notion of a mechanism is broad, and Harré emphasised that it is not restricted to "anything specifically mechanical": a "[c]lockwork is a mechanism, Faraday's strained space is a mechanism, electron quantum jumps is a mechanism, and so on" (2004, 4).

Models can also step in when theories are too complex to handle. This can happen, for instance, when the equations of the theory are mathematically intractable. In such cases, one can find a model that approximates the theory. As Redhead noted, this can be done in two ways (1980, 150–152): either one finds approximate solutions to exact equations or one finds an approximate equation that one can solve exactly. If one finds either an approximate solution or an approximate equation, these can be seen as approximate models of the theory. However, models can also step in when the relation between the model and the theory is not a clearly defined mathematical approximation. Hartmann (1999) discusses the case of quark confinement in elementary particle physics. The nucleus of atoms is made up of nucleons: protons and neutrons. Nucleons themselves are made up of quarks. How do quarks interact to form a stable nucleon? The general theory covering the behaviour of quarks is quantum chromodynamics. Unfortunately, the theory is too complicated to apply to protons. Computer simulations suggest that at low energies so-called quark confinement occurs, and quarks come together to form nucleons. This, however, leaves the nature of this confinement unexplained and poorly understood, with a number of different kinds of confinement possible and the theory unable to adjudicate between them. To fill in this gap, physicists constructed a phenomenological model, now known as the MIT bag model, which takes the main known features of the theory into account and fills the missing details with postulated configurations. According to the model, nucleons consist of three massive quarks that move freely in a rigid sphere of radius R, where the sphere guarantees that the quarks remain confined within the nucleon. This assumption is motivated by the basic theory, but it does not deductively follow from it. The model then allows for the calculation of the radius R and the total energy of the particle. In this way, the model yields results where the theory is silent, and it fills a gap that the theory leaves open.

6. Applying theories through models

Cartwright argues that models not only aid the application of theories that are somehow incomplete; she submits that models are always involved when a theory with an overarching mathematical structure is applied to a target system. The main theories in physics fall into this category: classical mechanics, quantum mechanics, electrodynamics, and so on. In fact, applying such theories involves two kinds of models: interpretative models and representative models.

Let us begin with *interpretative models*. Overarching mathematical theories like classical mechanics appear to provide general descriptions of a wide range of objects that fall within their scope. However, on closer inspection, it turns out that these theories do not apply to the world directly. The reason for this is that they employ abstract terms, i.e. terms that apply to a target system only if a description couched in more concrete terms also applies to the target. Cartwright offers the following two conditions for a concept to be abstract relative to another concept:

> First, a concept that is abstract relative to another more concrete set of descriptions never applies unless one of the more concrete descriptions also applies. These are the descriptions that can be used to "fit out" the abstract description on any given occasion. Second, satisfying the associated concrete description that applies on a particular occasion is what satisfying the abstract description consists in on that occasion. *(1999b, 39)*

She offers the example of *work*. Having responded to an email, having revised a section of a paper, and having attended a meeting is what my having done work this morning consists in. If I tell a friend over lunch what I have done and he responds, "well, you've responded to an email, revised a section, and attended a meeting, but when did you work?", he either does not understand the concept of work or, more likely, is joking with me.

Cartwright submits that important concepts that appear in mathematised theories are abstract in the same way as *work*. The concept of *force*, for instance, is abstract in that it applies only if a more concrete concept also applies. There is no such thing as "nothing but a force" acting on a body. There being a force between two bodies on a particular occasion consists in them gravitationally attracting each other, or electrostatically repelling each other, or … These more concrete claims fit out the abstract claim of there being a force. Force, therefore, is an abstract property and "Newton's law tells that whatever has this property has another, namely having a mass and an acceleration which, when multiplied together, give the […] numerical value, *F*" (1999b, 43). Force, therefore, has no independent existence; it exists only in its more specific forms like gravity, electrostatics, and so on. Specifying what concrete claims fit out abstract claims amounts to specifying an *interpretative model*. An interpretative model then consists of the "actors" that fit out the abstract claims of the theory.

Let us now turn to *representative models*. Cartwright regards representative models as ones that are built to "represent real arrangements and affairs that take place in the world" (1999b, 180). These models have two crucial features. The first is that they are highly idealised. Constructing a representative model involves twisting and distorting the properties of the target in many ways and the result of this process is in no way a mirror image of the target. Indeed, Cartwright notes that "it is not essential that the models accurately describe everything that actually happens; and in general it will not be possible for them to do so" (1983, 140). Second, all these distortions notwithstanding, the model still is a representation of the target, albeit one that is inaccurate in certain respects. The principles of the theory therefore apply to "highly fictionalized objects" (1983, 136) in the representational model. So, one has to distort reality to force it into the corset of the theory: "our prepared descriptions lie" because "in general we will have to distort the true picture of what happens if we want to fit it into the highly constrained structures of our mathematical theories" (139). Without these distortions, the theory would be inapplicable.

We are now in a position to see how the two notions of an interpretative model and a representational model work together in the application of a theory to a real-world target. To apply a theory, scientists must construct a model. This model must be such that it is, at once, an interpretative model of the general theory at hand (which means that it is couched in terms of concepts that fit out the abstract concepts of the theory) and a representative model of the target system (which means that it stands in a certain representational relation to the target).

7. Models as mediators

The relation between models and theories can be even looser than in the cases we have discussed so far. The contributors to a programmatic collection of essays edited by Morgan and Morrison (1999b) rally around the idea of "models as mediators", and so it is apt to call the vision of modelling that emerges from this project the *Models as Mediators View*. This view sees models as instruments that mediate between theories and the world while remaining independent from both. Models are, therefore, as Morgan and Morrison put it, "autonomous agents" (1999a, 10). The autonomy of models has four dimensions: construction, functioning, representing, and learning (10–12). Let us look at each of these in turn.

The first and most important dimension is independence in construction. Morgan and Morrison observe that "model construction is carried out in a way which is to a large extent independent of theory" (1999a, 13), and Morrison locates models as being "between physics and the physical world" (1998, 65). This is because "theory does not provide us with an algorithm from which the model is constructed and by which all modelling decisions are determined" (Morgan and Morrison 1999a, 16). In her contribution to the collection, Cartwright portrays the Semantic View of theories as a "vending machine" view of model construction:

> The theory is a vending machine: you feed it input in certain prescribed forms for the desired output; it gurgitates for a while; then it drops out the sought-for representation, plonk, on the tray, fully formed, as Athena from the brain of Zeus. This image of the relation of theory to the models we use to represent the world is hard to fit with what we know of how science works. Producing a model of a new phenomenon such as superconductivity is an incredibly difficult and creative activity.
>
> *(1999b, 247)*

According to Cartwright, the "vending machine view" of theories is wrong on at least two counts. First, it erroneously assumes that all ingredients that are needed for the construction of a model are already contained in the theory. As we have seen in the previous section, she sees representative models as an essential ingredient for the application of a theory. The construction of such a model requires resources that go beyond what theories can offer. Discussing quantum models of superconductivity, Cartwright notes that theories leave out much of what is needed to produce a model capable of generating an empirical prediction. While theories contain general principles, they contain no information either about the real materials from which a superconductor is built or about the various approximation schemes and the mathematical techniques needed to handle them. Second, the view is wrong in assuming that models embody only one theory. The internal setup of a model is often a complicated conglomerate of elements from different theories. Cartwright illustrates this point with the Ginzburg–Landau model of superconductivity (1999a, 244–245), but the point also holds about other models like the classical London model of superconductivity (Suárez 1999) and models of business cycles (Boumans 1999). The same is also true of contemporary climate models which incorporate elements from different theories, including mechanics, fluid dynamics, electrodynamics, quantum theory, chemistry, and biology (Frigg, Thompson, and Werndl 2015). Models of this kind do not belong to a family of models that form a theory in anything like the way that the Semantic View posits; in fact, they do not belong to any particular theoretical framework at all.

The second dimension of autonomy is functioning: models can perform many functions without relying on theories. One of these functions is to aid theory construction (Morgan and Morrison 1999a, 18). As we have seen previously, models can play a role in theory construction (Section 3) and in exploring theories (Section 4), which they can do only if they are autonomous from theories. Models also serve as a means for policy intervention (Morgan and Morrison 1999a, 24). Central banks use economic models to inform monetary policy decisions, for instance, whether to change the base rate, and models can do this independently from theory.

Representation is the third dimension of autonomy. Morgan and Morrison point out that the "critical difference between a simple tool and a tool of investigation is that the latter involves some form of representation: models typically represent either some aspect of the world, or some aspect of our theories about the world, or both at once" (1999a, 11). They emphasise that representing does not presuppose that there is "a kind of mirroring of a phenomenon, system or theory by a model" because representing is in no way tantamount to producing a copy, or effigy, of the target.[12]

The final dimension of autonomy is learning. Morgan and Morrison point out that we learn from models and argue that this happens in two places: in building the model and in manipulating it (1999a, 11–12). As we have seen earlier in this section, there are no general rules or algorithms for model building and hence insights gained into what fits together and how during the process of construction are invaluable sources for learning about the model (30–31). The second place to learn about the model is when we manipulate it. Morgan (1999) notes that Fisher did not find out about the properties of his monetary models by contemplating them, but by manipulating them to show how the various parts of the model work together to produce certain results.

8. Separating models from theories

So far, we worked under the assumption that models and theories are clearly distinct, and we focussed on the relation between them. In practice, this is not always a realistic assumption. In fact, in some cases it is not clear where the line between them should be drawn, and whether something is a model or a theory. An example is Bohr's account of the atom, which is sometimes referred to as the "Bohr model" and sometimes as the "Bohr theory" of the atom. This problem not only besets philosophical analysis; it also arises in scientific practice. Bailer-Jones interviewed a group of nine physicists about their understanding of models and their relation to theories. She reports that the following views were expressed (2002, 293):

1 There is no real difference between model and theory.
2 Models turn into theories once they are better and better confirmed.

3 Models contain necessary simplifications and deliberate omissions, while theories are the best we can do in terms of accuracy.
4 Theories are more general than models. Modelling becomes a case of applying general theories to specific cases.

The first suggestion is too radical to do justice to many aspects of practice, where a distinction between models and theories is clearly made. The second view is encapsulated in phrases like "it's just a model", which indicate either that scientists take a cautious attitude towards a certain proposition that they regard as speculative or provisional, or that something is known to be false and entertained only for heuristic purposes. But, models and theories are not distinguished by their degree of confirmation. There can be well-confirmed models and unconfirmed theories. The third proposal is up to something, but it ultimately does not hold water. It is true that models involve idealisations and omissions of all kinds, but so do theories. Newtonian mechanics, for instance, deals with point masses that move in a Euclidean space, and it omits most properties of the objects in its target domain (it omits, for instance, colour, temperature, and chemical constitution of its targets) but that does not seem to strip Newtonian mechanics of its status as a theory.

The fourth suggestion is closely aligned with a view that has emerged in the literature on models. In the wake of the debates we have reviewed in this chapter, models have become the focal point of attention and the emphasis has shifted so far away from theories that Morrison detects the need for a "redress of the imbalance" (2007, 195). She asks "where have all the theories gone" and then sets out to articulate how theories are different from models. Morrison points out that models contain a great deal of "excess" structure like approximation methods, mathematical techniques, and highly stylised descriptions of certain parts of the target, and she notes that one would not want to count these as part of a theory (197). This can be avoided if "theory" is reserved for a "theoretical core", which contains the constitutive assumptions of the theory. In the case of Newtonian mechanics, the core consists of the three laws of motion and the law of universal gravitation (197), in the case of classical electrodynamics of Maxwell's equations, in the case of relativistic quantum mechanics of the Dirac equation (205), and in the case of quantum mechanics of the Schrödinger equation (214). The core of a theory constrains the behaviour of objects that fall within the scope of the theory, and it plays a crucial role in the construction of models. Models concretise the abstract laws of the theory and put them to use by adding elements that are specific to the situation. In this way, theories assist the construction of models without determining the way in which they are built. Models are specific in that they are adapted to a particular situation and a particular problem, while the theories on which they are based contain the general principles of wide scope.

The problem with the "theoretical core" view of theories as presented by Morrison is that the notion of a theoretical core is introduced through examples – Newton's laws of motion, Maxwell's equations, and so on – and is then not further analysed. Morrison seems to regard this as an advantage when she observes that "nothing about this way of identifying theories requires that they be formalized or axiomatized" (2007, 205). However, this pragmatism must seem unsatisfactory to those who have contributed to the development of the two grand views of theories and who will feel that we have now come full circle. Neither the Syntactic View nor the Semantic View would disagree that what makes a theory a theory is a theoretical core. The question they are concerned with is how this notion can be analysed and what kind of objects theoretical principles are. This question is left open.

9. Conclusion

We have discussed a number of different relationships between models and theories that can be found in the practice of science. These range from complete independence to total dependence, and many things in between. Many of these cases do not seem to sit well either with the Syntactic View or with the Semantic View, and they show that there is nothing like "the" relation between models and theories.

Notes

1 Sections 3–8 of this chapter are based on Chapter 13 of my (2023).
2 I note that the label "Syntactic View" is a misnomer because it gives the mistaken impression that the view only deals with the syntax of theories. Some readers may object to calling the Syntactic View an orthodoxy because it has been superseded by the Semantic View long ago. This narrative has become untenable in the last decade, when the Syntactic View had a veritable revival. For a discussion, see, for instance, Halvorson (2016).
3 The exact form of correspondence rules has been the subject matter of extensive debates. For a survey, see, for instance, Percival (2000).
4 For a detailed discussion of the problems faced by both the Syntactic View and the Semantic View, see Chapters 1–8 of my (2023) and references therein.
5 The locus classicus for a discussion of analogies is Hesse (1963). For further discussions of analogies and analogical models, see Chapter 10 of my (2023) and references therein.
6 For a discussion, see, for instance, Friedman (1983).
7 The model was formulated by Lotka (1925) and Volterra (1926). Kingsland (1985, chap. 5) gives a historical account of the development of the model. For philosophical discussions, see, for instance, Knuuttila and Loettgers (2017) and Weisberg and Reisman (2008).
8 For basic introductions to chaos and discussions of its philosophical ramifications, see Kellert (1993) and Smith (1998). Argyris, Faust and Haase (1994) and Tabor (1989) offer advanced discussions. Parker (1998) discusses the question of whether it was really Poincaré who discovered chaos.
9 For instance, the dynamics of KAM type systems near a hyperbolic fixed point can be modelled by the baker's transformation. For a discussion, see Berkovitz, Frigg, and Kronz (2006, 680–687).
10 For a discussion of quantum field theory, see, for instance, Ruetsche (2011).
11 Apostel does not provide an example. I am grateful to Julian Reiss for suggesting the quantity theory of money to me. For a discussion of the theory, see Humphrey (1974).
12 For a discussion of how models represent their targets, see Frigg and Nguyen (2020) and Nguyen and Frigg (2022).

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